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Aircraft Systems Technical Memorandum 106

AcSim - AIRCRAFT SIMULATION PROGRAM
WITH APPLICATION TO FLIGHT PROFILE GENERATION (U)

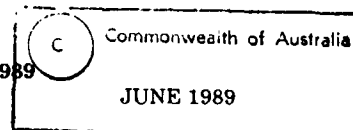
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A.G. PAGE

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Aircraft Systems Technical Memorandum 106

**AcSim - AIRCRAFT SIMULATION PROGRAM
WITH APPLICATION TO FLIGHT PROFILE GENERATION**

by

A.G. Page

SUMMARY

Program AcSim was designed to be used in conjunction with a flight profile generator program (FPG2) which is used with an Inertial Navigation System simulator. Both AcSim and FPG2 were written in FORTRAN-77 on an ELXSI System 6400. AcSim is implemented as a subroutine of FPG2 and is called if the user selects a dynamic manoeuvre segment to be performed. AcSim calculates the dynamic state of an aircraft for a sequence of user-specified control surface deflections. The aircrafts linear accelerations and angular velocities are sent back to FPG2 for subsequent processing.

A six-degree-of-freedom model is used to simulate the user-defined aircraft. AcSim implements a State Vector approach to solve the non-linear equations of motion simultaneously using a 4th order Runge-Kutta scheme. An optional "Eigen Analysis" can be performed which provides the user with longitudinal and lateral characteristic equations, their roots (eigen values), transfer functions and unit impulse response functions for the given flight condition.



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NOMENCLATURE

Symbol	Definition
$\hat{a}_x, \hat{a}_y, \hat{a}_z$	Non-dimensional linear accelerations, $d\hat{u}/d\tau, d\hat{v}/d\tau, d\hat{w}/d\tau$
C_L	Aircraft lift coefficient, $2mg \cos \Theta_0 / \rho V_0^2 S$
g	Gravity
$I_x = A$	Moment of inertia of the aircraft about Ox
$I_y = B$	Moment of inertia of the aircraft about Oy
$I_z = C$	Moment of inertia of the aircraft about Oz
$I_{xy} = D$	Product of inertia of the aircraft about Oxy
$I_{xz} = E$	Product of inertia of the aircraft about Oxz
$I_{yz} = F$	Product of inertia of the aircraft about Oyz
i_A	Non-dimensional moment of inertia about x-axis, A/ms^2
i_B	Non-dimensional moment of inertia about y-axis, B/ml_T^2
i_C	Non-dimensional moment of inertia about z-axis, C/ms^2
i_E	Non-dimensional product of inertia about Oxz , E/ms^2
k	$C_L/2$
k'	$-k \tan \Theta_0$
L	Moment about x-axis due to external forces
L_p, L_r, L_w	Rolling moment derivatives,
L_ξ, L_ζ	$\partial L/\partial p, \partial L/\partial r, \partial L/\partial v, \partial L/\partial \xi, \partial L/\partial \zeta$
ℓ_T	Tailplane lever arm (aircraft c.g. to tailplane a.c.)
l_p	Non-dimensional rolling moment derivative due to rate of roll, $L_p/\rho V_0 S s^2$
l_r	Non-dimensional rolling moment derivative due to rate of yaw, $L_r/\rho V_0 S s^2$
l_v	Non-dimensional rolling moment derivative due to sideslip, $L_v/\rho V_0 S s$
l_ξ	Non-dimensional rolling moment derivative due to ailerons, $L_\xi/\rho V_0^2 S s$
l_ζ	Non-dimensional rolling moment derivative due to rudder, $L_\zeta/\rho V_0^2 S s$
M	Moment about y-axis due to external forces
M_q, M_u, M_w	Pitching moment derivatives,
$M_{\dot{w}}, M_\eta$	$\partial M/\partial q, \partial M/\partial u, \partial M/\partial w, \partial M/\partial \dot{w}, \partial M/\partial \eta$
m	Aircraft mass
m_q	Non-dimensional pitching moment derivative to rate of pitch, $M_q/\rho V_0 S \ell_T^2$
m_u	Non-dimensional pitching moment derivative due to velocity increment along Ox , $M_u/\rho V_0 S \ell_T$
m_w	Non-dimensional pitching moment derivative due to velocity increment along Oz , $M_w/\rho V_0 S \ell_T$
$m_{\dot{w}}$	Non-dimensional pitching moment derivative due to rate of change of w , $M_{\dot{w}}/m \ell_T$
m_η	Non-dimensional pitching moment derivative due to elevators, $M_\eta/\rho V_0^2 S \ell_T$
m'_q, m'_u, m'_w	$m_q/i_B, m_u/i_B, m_w/i_B$
$m'_{\dot{w}}, m'_\eta$	$m_{\dot{w}}/i_B, m_\eta/i_B$
N	Moment about z-axis due to external forces
N_p, N_r, N_v	Rolling yawing derivatives,
N_ξ, N_ζ	$\partial N/\partial p, \partial N/\partial r, \partial N/\partial v, \partial N/\partial \xi, \partial N/\partial \zeta$
n_p	Non-dimensional yawing moment derivative due to rate of roll, $N_p/\rho V_0 S s^2$
n_r	Non-dimensional yawing moment derivative due to rate of yaw, $N_r/\rho V_0 S s^2$
n_v	Non-dimensional yawing moment derivative due to sideslip, $N_v/\rho V_0 S s$
n_ξ	Non-dimensional yawing moment derivative due to ailerons, $N_\xi/\rho V_0^2 S s$
n_ζ	Non-dimensional yawing moment derivative due to rudder, $N_\zeta/\rho V_0^2 S s$

.../cont.

NOMENCLATURE (cont.)

Symbol	Definition
p	Angular velocity of $Oxyz$ about Ox (roll rate), $d\phi/dt$
\dot{p}	Rate of change of roll rate, dp/dt
\hat{p}	Non-dimensional angular velocity about Ox , $d\phi/d\tau$
$\dot{\hat{p}}$	Non-dimensional angular acceleration about Ox , $d^2\phi/d\tau^2$
q	Angular velocity of $Oxyz$ about Oy (pitch rate), $d\theta/dt$
\dot{q}	Rate of change of pitch rate, dq/dt
\hat{q}	Non-dimensional angular velocity about Oy , $d\theta/d\tau$
$\dot{\hat{q}}$	Non-dimensional angular acceleration about Oy , $d^2\theta/d\tau^2$
r	Angular velocity of $Oxyz$ about Oz (yaw rate), $d\psi/dt$
\dot{r}	Rate of change of yaw rate, dr/dt
\hat{r}	Non-dimensional angular velocity about Oz , $d\psi/d\tau$
$\dot{\hat{r}}$	Non-dimensional angular acceleration about Oz , $d^2\psi/d\tau^2$
S	Wing planform area
s	Wing semi-span
∇	Laplace operator
t	Time
\hat{t}	Time non-dimensionalisation factor, $m/\rho V_0 S$
U	Component of aircrafts center of gravity velocity along Ox
U_0	Component of aircrafts c.g velocity along Ox in the initial steady state flight
u	Velocity perturbation along x-axis, $U - U_0$
\hat{u}	Non-dimensional velocity perturbation along x-axis, u/V_0
V	Component of aircrafts center of gravity velocity along Oy
V_0	Initial flight speed
v	Velocity perturbation along y-axis
\hat{v}	Non-dimensional velocity perturbation along y-axis, v/V_0
W	Component of aircrafts center of gravity velocity along Oz
W_0	Component of aircrafts c.g velocity along Oz in the initial steady state flight
w	Velocity perturbation along z-axis, $W - W_0$
\hat{w}	Non-dimensional velocity perturbation along z-axis, w/V_0
X	External force acting on aircraft along x-axis
$X_q, X_u,$ X_w, X_η	Ox Force component derivatives, $\partial X/\partial q, \partial X/\partial u, \partial X/\partial w, \partial X/\partial \eta$
x_q	Non-dimensional Ox force component derivative due to rate of pitch, $X_q/\rho V_0 S \ell_T$
x_u	Non-dimensional Ox force component derivative due to velocity increment along Ox , $X_u/\rho V_0 S$
x_w	Non-dimensional Ox force component derivative due to velocity increment along Oz , $X_w/\rho V_0 S$
x_η	Non-dimensional Ox force component derivative due to elevators, $X_\eta/\rho V_0^2 S$
Y	External force acting on aircraft along y-axis
$Y_p, Y_r, Y_n,$ Y_ξ, Y_ζ	Side force derivatives, $\partial Y/\partial p, \partial Y/\partial r, \partial Y/\partial v, \partial Y/\partial \xi, \partial Y/\partial \zeta$
y_p	Non-dimensional side force derivative due to rate of roll, $Y_p/\rho V_0 S s$
y_r	Non-dimensional side force derivative due to rate of yaw, $Y_r/\rho V_0 S s$
y_n	Non-dimensional side force derivative due to sideslip, $Y_n/\rho V_0 S$

.../cont.

NOMENCLATURE (cont.)

Symbol	Definition
y_ξ	Non-dimensional side force derivative due to ailerons, $Y_\xi/\rho V_0^2 S$
y_ζ	Non-dimensional side force derivative due to rudder, $Y_\zeta/\rho V_0^2 S$
Z	External force acting on aircraft along z-axis
$Z_q, Z_u,$ Z_w, Z_η	0z Force component derivatives, $\partial Z/\partial q, \partial Z/\partial u, \partial Z/\partial w, \partial Z/\partial \eta$
z_q	Non-dimensional Oz force component derivative due to rate of pitch, $Z_q/\rho V_0 S \ell_T$
z_u	Non-dimensional Oz force component derivative due to velocity increment along Ox, $Z_u/\rho V_0 S$
z_w	Non-dimensional Oz force component derivative due to velocity increment along Oz, $Z_w/\rho V_0 S$
z_η	Non-dimensional Oz force component derivative due to elevators, $Z_\eta/\rho V_0^2 S$
Greek Symbols	
η	Angular displacement of elevator
μ_1	Longitudinal relative density parameter, $m/\rho S \ell_T$
μ_2	Lateral relative density parameter, $m/\rho S s$
τ	Non-dimensional time, t/\bar{t}
ϕ	Roll angle
ψ	Yaw angle
θ	Pitch angle
Θ_0	Initial pitch attitude
ρ	Air density
ξ	Angular displacement of ailerons
ζ	Angular displacement of rudder

1. INTRODUCTION

Inertial navigation plays an important role in air vehicle navigation due to its high accuracy over short periods of time, good performance in a high dynamic environment, total independence of external systems, and its immunity to jamming. This is why, even with the advent of GPS Navstar (Global Positioning System Navigation Satellite Tracking And Ranging) the Inertial Navigation System (INS) is likely to remain a primary navigation system with GPS, or some other system, continually calibrating the INS.

An inertial measurement unit is very sensitive to errors in its components and their calibration. The resulting navigation errors grow with time. Because different error sources (accelerometer bias, gyro drift, mis-alignment, etc.) can produce navigation errors with similar characteristics, INS simulations are necessary to study the effects of individual error sources. It is also extremely expensive to test INS's by flight trials.

A simulated INS requires simulated sensor inputs, so the aircraft's dynamic environment must be emulated. A mathematical aircraft model is required to obtain a time history of the aircraft's accelerations.

A program exists, FPG2 (ref. 1), which creates flight profiles from a selection of four 'ideal' manoeuvres. 'Ideal' refers to the fact that the aircraft's state is determined using simple equilibrium equations rather than the flight dynamic equations. As well as determining the dynamic environment (linear and angular accelerations) of the INS, FPG2 also calculates the nominally true, or reference, position and velocity of the aircraft over the Earth reference ellipsoid.

With ideal manoeuvres, the aircraft proceeds through the flight by executing sequences of rotations and linear accelerations, which FPG2 calculates from the specifications of each manoeuvre. An ideal manoeuvre is specified by its type and duration, and some of the following: heading or pitch change, linear acceleration along path, maximum allowed z-axis (See Figure 1 for aircraft axes) acceleration, maximum bank angle, maximum angular velocity, and the angular acceleration about the relevant axis.

In order to improve the fidelity of the FPG2-generated dynamic environment of the aircraft (and hence the INS), it is necessary to include in the simulation a mathematical model representing the six-degree-of-freedom dynamic model of the specific aircraft. This document describes the aircraft model and computer program (AcSim) used for this purpose.

AcSim uses a standard six-degree-of-freedom aircraft model with pilot (user) inputs via the control surfaces; elevators, ailerons and rudder. A State-Vector approach was adopted to solve the non-linear equations of motion simultaneously using a 4th order Runge-Kutta scheme. This scheme was implemented to compliment the 4th order Runge-Kutta scheme used in FPG2. An optional, so-called 'Eigen Analysis', is also available which provides the user with the longitudinal and lateral characteristic equations, eigen values, transfer functions, and unit impulse response functions. This analysis enables the user to identify the stable and unstable modes of the aircraft and provides an indication of the degree of stability or instability.

Because FPG2 was written in FORTRAN-77 and AcSim was written specifically as an extension to FPG2, AcSim was also written in FORTRAN-77. AcSim is implemented as a subroutine of FPG2 and may be selected by the FPG2 user at any time during the simulated flight, as an alternative to any of the four 'ideal' manoeuvres. If AcSim is called, the aircraft's linear

acceleration and angular velocity response to the user's control surface inputs are determined by AcSim and returned to FPG2 for further processing.

2. PROGRAM SUMMARY

The ideal manoeuvres of FPG2 are totally independent of the particular aircraft, whereas AcSim requires that the aircraft be fully defined (See Appendix 1). If AcSim is called the user must enter the aircraft data, either interactively or via a file.

Once the aircraft and its flight condition are defined the user is asked if an 'Eigen Analysis' is desired. The eigen analysis consists of determining the longitudinal and lateral characteristic equations and their roots (eigen values). The longitudinal eigen values consist of the Short Period Oscillation and Long Period (Phugoid) Oscillation modes, and the lateral eigen values consist of the Roll, Spiral, and Dutch Roll modes. The type (real or complex), sign (positive or negative), and magnitude of the modes give the user valuable information about the dynamic stability characteristics of the defined aircraft at the specified flight condition. After the eigen values have been determined the user is given the option of having the transfer functions and unit impulse response functions determined for each of the control surfaces.

Even if the user doesn't request the eigen analysis, the eigen values are still calculated. If, and only if, an atypical situation occurs (for example, an unstable SPO mode) the user is notified.

After the eigen analysis is completed the user is required to enter the control surface deflection schedule for the current sequence of manoeuvres. A so-called 'manoeuvre sequence' consists of one or more control surfaces (elevators, ailerons, rudder) being deflected. The manoeuvre is considered finished when the time duration for required response output (user-specified) to the current manoeuvre control deflections has elapsed. This data can be entered at any of three levels, depending on whether the user has saved manoeuvre data files from previous runs:

- 1) Define control deflection curve. Control deflection curves are specified by as many data pairs (magnitude of deflection, time) as necessary to approximate the desired control deflection curve. This data may be saved in a user-named Uninterpolated-Manoeuvre-Sequence-File (unMSF)
- 2) Input previously saved unMSF.
Once the control deflection curve has been specified, via 1) or 2), the deflection curve is assumed linear between data pairs and is interpolated so that data points are obtained at every time increment, Δt . The interpolated data may be saved in a user-named Manoeuvre-Sequence-File (MSF).
- 3) Input previously saved MSF.

The user can input as many manoeuvres as desired until the segment duration, specified in FPG2, has elapsed. Dynamic manoeuvres (AcSim) is one out of a selection of five segment types available in FPG2.

When AcSim has the aircraft data and interpolated control deflection data available, the aircraft response can be calculated. The linear accelerations and angular velocities are subsequently returned to FPG2 at each time increment. The timing is controlled by FPG2.

Figure 2 shows the program summary flowchart of AcSim. Figure 3 shows the structure of subroutines for AcSim. Figure 4 is the group of subroutines which performs the eigen analysis.

3. PROGRAM THEORY

The general dynamical equations for a rigid aircraft in body axes are (p12, ref. 3) :

$$m(\dot{U} - rV + qW) = X \quad (1)$$

$$m(\dot{V} - pW + rU) = Y \quad (2)$$

$$m(\dot{W} - qU + pV) = Z \quad (3)$$

$$I_x \dot{p} - (I_y - I_z)qr - I_{yx}(q^2 - r^2) - I_{xz}(\dot{r} + pq) - I_{xy}(\dot{q} - rp) = L \quad (4)$$

$$I_y \dot{q} - (I_z - I_x)rp - I_{zy}(r^2 - p^2) - I_{xy}(\dot{p} + qr) - I_{yz}(\dot{r} - pq) = M \quad (5)$$

$$I_z \dot{r} - (I_x - I_y)pq - I_{xz}(p^2 - q^2) - I_{xy}(\dot{q} + rp) - I_{yz}(\dot{p} - qr) = N \quad (6)$$

The aircraft is acted upon by the external forces, X, Y, Z along Ox, Oy, Oz respectively, and the moments of the external forces, about Ox, Oy, Oz are L, M, N (see Figure 1).

Note that the initial flight speed vector is made coincident with Ox , so that the aircraft's co-ordinate system is defined by a set of stability axes.

Also note that the notation used in this document and in the AcSim source code for the aerodynamic derivatives etc., is the British system (used in ref. 3). A comparison between this system and the American system is given on p. xxi ref. 3.

3.1 Eigen Analysis

The 'Eigen Analysis' utilises the classical analytical dynamic stability analysis approach. The above six non-linear differential equations are linearised (on the assumption that any disturbances from steady trimmed rectilinear motion are small) and non-dimensionalised. Because the longitudinal and lateral small disturbance equations are uncoupled (i.e. longitudinal and lateral motion are independent) two sets of three equations are obtained. The characteristic equations and their roots (i.e. eigen values) are determined for both the longitudinal and lateral sets of equations. The longitudinal and lateral transfer functions and unit impulse response functions for inputs from each of the three control surfaces are also determined upon user request. If the user doesn't request the eigen analysis to be performed then only the eigen values for the system are determined and any non-normal (meaning not the type of result expected for a conventional aircraft) conditions are reported to the user. The theory for this eigen analysis is given in **Appendix 2**.

3.2 Aircraft Response Determination

A State-Vector approach was adopted to solve the six non-linear equations of motion simultaneously using a fourth order Runge-Kutta scheme.

The state variables were selected as U, V, W and p, q, r . Therefore the State Vector is:

$$S\dot{V} = \begin{bmatrix} U \\ V \\ W \\ p \\ q \\ r \end{bmatrix}$$

The six non-linear differential equations of motion (1-6) are re-arranged into a form suitable for implementing a Runge-Kutta numerical solving scheme (Ref. 5 and 6): (Note that the mass distribution is assumed constant, i.e. $d(\text{mass, inertial})/dt = 0$, and symmetrical about the roll axis, i.e. $I_{xy}, I_{yz} = 0$)

$$S\dot{V} = \begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} rV - qW + \frac{X}{m} \\ pW - rU + \frac{Y}{m} \\ qU - pV + \frac{Z}{m} \\ A_1qr + A_2pq + A_3N + A_4L \\ B_1rp + B_2p^2 + B_3r^2 + B_4M \\ C_1pq + C_2qr + C_3N + C_4L \end{bmatrix}$$

where

$$A_1 = C_1 = f(I_x, I_y, I_z, I_{xz})$$

The forces (X, Y, Z) and moments (L, M, N) acting on the aircraft must be determined before the equations of motion can be solved for the aircrafts response.

The external forces acting on the aircraft consist of aerodynamic and gravitational components, whereas the moments are only aerodynamic (since the axis origin is at the aircraft's centre of gravity).

The aerodynamic components are determined using linear aerodynamics and the aerodynamic derivatives are assumed constant. Non-linear aerodynamics and/or non-constant aerodynamic derivatives could be incorporated if a special need arose, but for the intended application both are unnecessary complications which would require a much more detailed aerodynamic knowledge of the aircraft.

Appendix 3 details the determination of the forces and moments acting on the aircraft.

The Runge-Kutta scheme to solve the equations of motion is implemented as detailed in **Appendix 4**.

The linear accelerations and angular velocities are then returned to FPG2 for further processing.

4. PROGRAM VALIDATION

Validation of the AcSim source code was carried out by first validating the 'Eigen Analysis' section of the program.

Using data available for a Beechcraft Bonanza (see **Figure 5** for the data required by AcSim file summary) the AcSim Eigen Analysis was performed. Part of the result file from this analysis is reproduced in **Figure 6**. The characteristic equations, eigen values, transfer functions, and unit impulse response functions (for all 3 control surfaces) were verified as correct by manual calculation of the results by the author in previous studies.

To validate the Runge-Kutta response solution of the non-linear equations of motion, some test runs of the program were conducted. Each control surface was deflected separately using the schedule depicted in **Figure 7**. To enable some comparison, a numerical convolution scheme was applied to the unit impulse response functions to obtain the aircraft response solution to the linear small-disturbance equations of motion (see **Appendix 5**). Test runs identical to those performed on AcSim were conducted. The resulting aircraft attitude responses for both solutions are shown in **Figures 8a-e**. As

can be seen from these plots, the results agree quite well, with variations attributable to differences between linear and non-linear solutions.

5. ASSUMPTIONS IN ACSIM

AcSim makes assumptions (or rather, approximations) about the aircraft itself, which are :

1. It has a longitudinal plane of symmetry, this being the vertical plane through the centre line of the fuselage. The mass distribution of the aircraft is assumed symmetrical with respect to this plane, so that

$$I_{xy} = 0 \quad \text{and} \quad I_{yz} = 0$$

Note that I_{xz} will not be zero, unless Ox and Oz are principal axes of inertia.

2. The mass distribution of the aircraft is assumed constant, so that the aircraft mass and inertias are invariant.
3. The aircraft is assumed to be a rigid body, that is, any distortion of the structure is ignored (i.e no aeroelastic effects).

Linear aerodynamics is assumed by AcSim, and the aerodynamic derivatives are assumed constant. This was done to avoid complications and to minimise the amount of aerodynamic data required to model the aircraft.

6. CONCLUDING REMARKS

It should be stressed that program AcSim is implemented as a subroutine of FPG2. AcSim is an extension to FPG2 to provide the capability, if required, of a more realistic dynamic aircraft response. The AcSim program structure and the implementation of a 4th order Runge-Kutta scheme were written explicitly to compliment FPG2.

AcSim could be made into a stand-alone aircraft simulation model, but some modification to the code would be necessary. In particular, the aircraft attitude, which is currently passed into AcSim from FPG2 as a direction cosine matrix, would have to be computed in AcSim. Also, some modification to the program structure and timing would be required.

The AcSim source code validation presented in this document does lend credibility to AcSim results, however a non-linear solution to further validate the code will be forthcoming. This will probably constitute an A.C.S.L. (Advanced Continuous Simulation Language) model.

AcSim could be developed in the future to incorporate such things as; non-linear aerodynamics, non-constant aerodynamic derivatives, or a manoeuvre auto-pilot. Currently, AcSim requires the user to fly the aircraft by specifying a series of control surface deflections. This makes it difficult, without many trials, for the user to successfully fly a desired flight profile. To overcome this handling problem, some kind of manoeuvre specification routine (auto-manoeuve pilot) could be set up to provide the user with the option of flying by deflecting controls or by manoeuvre specifications. The manoeuvre specifications would need to comprise such items as; heading or pitch change, maximum roll or pitch angle, maximum linear and angular accelerations, etc. The relevant controls would then be deflected to obtain the maximum specified accelerations and angles, and then deflected in the opposite sense to reach a steady condition at approximately the desired aircraft state. This type of manoeuvre-specifying flying could easily be incorporated

into the structure of AcSim by using the manoeuvre specifications to determine the required control deflection curves. The manual input of the control deflection curves would then be by-passed. Once the control deflection curves are input or created, the rest of AcSim would run as usual. Work in the field of manoeuvre auto-pilots is currently being done at ARL, and could be incorporated into AcSim if the need arose to develop AcSim further. At the moment no future development of AcSim is being planned.

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APPENDIX 1 : Aircraft Data required by AcSim

The program requires data to fully define the aircraft and its flight condition. The program will ask what units (SI or Imperial) the user is using, and so all subsequent data must be consistent with the specified units.

To define the flight condition the following data is required :

Air density, ρ (slugs/ft³ or kg/m³)

Flight speed, V_0 (ft/sec or m/s)

Initial attitude (pitch angle with respect to Earth), Θ_0 (degrees)

To define the aircraft, the following data is required :

Mass of aircraft, m (lb or kg)

Wing planform area, S (ft² or m²)

Longitudinal data :

Moment of inertia, I_{yy} or B (slugs.ft² or kg.m²)

Tail lever arm (aircraft c.g to tailplane a.c), l_T (ft or m)

Longitudinal aerodynamic derivatives (dimensional or non-dimensional):

x_u, x_w, x_q, x_η

z_u, z_w, z_q, z_η

$m_u, m_w, m_{\dot{w}}, m_q, m_\eta$

Lateral data :

Moments of inertia I_{xx} , I_{zz} , and I_{xz} or A , C , and E (slugs.ft² or kg.m²)

Wing span, b (ft or m)

Lateral aerodynamic derivatives (dimensional or non-dimensional):

$y_u, y_p, y_r, y_\xi, y_\zeta$

$l_u, l_p, l_r, l_\xi, l_\zeta$

$n_u, n_p, n_r, n_\xi, n_\zeta$

APPENDIX 2 : Eigen Analysis

A2.1 Equations of Motion

The six non-linear differential equations of motion are linearised and non-dimensionalised as given below. Because the longitudinal and lateral equations of motion are uncoupled we obtain three longitudinal and three lateral equations. The equations are linearised on the assumption that the disturbances from steady trimmed rectilinear motion are small.

Longitudinal Equations of Motion :

$$\begin{pmatrix} s - x_u & -x_w & -\frac{x_A}{\mu_1}s + \frac{c_A}{2} \\ -z_u & s - z_w & (1 + \frac{z_A}{\mu_1})s + \frac{c_A}{2} \tan \Theta_0 \\ \mu_1 m'_u & -\mu_1 m'_w s - \mu_1 m'_w & s^2 - m'_q s \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{w} \\ \hat{\theta} \end{pmatrix} = \begin{pmatrix} x_\eta \\ z_\eta \\ \mu_1 m'_\eta \end{pmatrix} \eta$$

Lateral Equations of Motion :

$$\begin{pmatrix} s - y_v & -\frac{y_E}{\mu_2}s + k & (1 - \frac{y_L}{\mu_1})s + k' \\ -\frac{\mu_2}{\mu_1} l_v & s^2 - \frac{l_E}{\mu_1}s & -\frac{y_E}{\mu_1} s^2 - \frac{l_L}{\mu_1} s \\ \frac{\mu_2}{\mu_1} n_v & \frac{y_E}{\mu_1} s^2 - \frac{n_E}{\mu_1} s & s^2 - \frac{n_L}{\mu_1} s \end{pmatrix} \begin{pmatrix} \hat{v} \\ \hat{\phi} \\ \hat{\psi} \end{pmatrix} = \begin{pmatrix} y_\xi \\ \frac{\mu_2}{\mu_1} l_\xi \\ \frac{\mu_2}{\mu_1} n_\xi \end{pmatrix} \xi + \begin{pmatrix} y_\zeta \\ \frac{\mu_2}{\mu_1} l_\zeta \\ \frac{\mu_2}{\mu_1} n_\zeta \end{pmatrix} \zeta$$

A2.2 Characteristic Equations

The program calculates the characteristic equations (longitudinal and lateral) by evaluating the determinant of the matrix:

$$\begin{pmatrix} s + A & Bs + C & Ds + E \\ F & Qs^2 + Gs + H & Is^2 + Js + K \\ L & Ms^2 + Ns + O & s^2 + Ps \end{pmatrix}$$

The coefficients (A-Q) are set at the beginning of the subroutines (Long and Lat) to correspond to the relevant equations of motion.

Longitudinal Characteristic Equation :

$$\Delta_{l,m,q} = \det \begin{pmatrix} s - x_u & -x_w & -\frac{x_A}{\mu_1}s + \frac{c_A}{2} \\ -z_u & s - z_w & (1 + \frac{z_A}{\mu_1})s + \frac{c_A}{2} \tan \Theta_0 \\ -\mu_1 m'_u & -\mu_1 m'_w s - \mu_1 m'_w & s^2 - m'_q s \end{pmatrix}$$

$$= s^4 + AAs^3 + BBs^2 + CCs + DD$$

Lateral Characteristic Equation :

$$\Delta_{l,n} = \det \begin{pmatrix} s - y_v & -\frac{y_E}{\mu_2}s + k & (1 - \frac{y_L}{\mu_1})s + k' \\ -\frac{\mu_2}{\mu_1} l_v & s^2 - \frac{l_E}{\mu_1}s & -\frac{y_E}{\mu_1} s^2 - \frac{l_L}{\mu_1} s \\ \frac{\mu_2}{\mu_1} n_v & \frac{y_E}{\mu_1} s^2 - \frac{n_E}{\mu_1} s & s^2 - \frac{n_L}{\mu_1} s \end{pmatrix}$$

$$AA s^5 + BB s^4 + CC s^3 + DD s^2 + EE s$$

A2.3 Eigen Values

The roots of the characteristic equations are the eigen values of the system.

Longitudinal Eigen Values :

The roots of the longitudinal characteristic equation are then calculated using an iterative method (see Ref. 3, pp207-9).

For a 'conventional' aircraft we expect a solution of the form:

$$\Delta_{long} = (s - s_1)(s - s_2)(s - s_3)(s - s_4)$$

where,

$$s_1 = \sigma_s + i\omega_s$$

$$s_2 = s_1^* = \sigma_s - i\omega_s$$

$$s_3 = \sigma_p + i\omega_p$$

$$s_4 = s_3^* = \sigma_p - i\omega_p$$

where

subscript 's' denotes Short Period Oscillation

subscript 'p' denotes Phugoid Oscillation

'*' denotes a complex conjugate.

σ = non-dimensional damping factor

ω = non-dimensional frequency

Lateral Eigen Values :

The roots of the lateral characteristic equation are found using an iterative method (see Ref. 3, pp211-13).

For a 'conventional' aircraft we expect a solution of the form:

$$\Delta_{lat} = AAs(s - s_1)(s - s_2)(s - s_3)(s - s_3^*)$$

where,

$s_1 = \sigma_1$	Roll mode
$s_2 = \sigma_2$	Spiral mode
$s_3 = \sigma_3 + i\omega_3$	Dutch Roll mode

A2.4 Transfer Functions

Transfer functions are the analytical relationship of any parameter to the input of any other input. That is;

G_Y^X = Transfer Function relating any parameter, X , to an input, Y .

Longitudinal Transfer Functions :

$$G_\eta^u(s) = \frac{\Delta_\eta^u}{\Delta_{long}}, \quad G_\eta^w(s) = \frac{\Delta_\eta^w}{\Delta_{long}}, \quad G_\eta^\theta(s) = \frac{\Delta_\eta^\theta}{\Delta_{long}}$$

The numerator of the transfer functions are determined using Cramer's Rule : (see Ref 4. pp318-21)

$$\begin{aligned} \Delta_\eta^u &= \det \begin{pmatrix} x_\eta & -x_w & -\frac{x_u}{\mu_1}s + \frac{c_k}{2} \\ z_\eta & s - z_w & (1 + \frac{z_u}{\mu_1})s + \frac{c_k}{2} \tan \Theta_0 \\ \mu_1 m_\eta' & -\mu_1 m_w' s - \mu_1 m_u' & s^2 - m_\eta' s \end{pmatrix} \\ &= UAs^3 + UBs^2 + UCs + UD \\ \Delta_\eta^w &= \det \begin{pmatrix} s - x_u & x_\eta & -\frac{x_u}{\mu_1}s + \frac{c_k}{2} \\ -z_u & z_\eta & -(1 + \frac{z_u}{\mu_1})s + \frac{c_k}{2} \tan \Theta_0 \\ -\mu_1 m_u' & \mu_1 m_\eta' & s^2 - m_\eta' s \end{pmatrix} \\ &= WAs^3 + WBs^2 + WCs + WD \\ \Delta_\eta^\theta &= \det \begin{pmatrix} s - x_u & -x_w & x_\eta \\ -z_u & s - z_w & z_\eta \\ -\mu_1 m_u' & -\mu_1 m_w' s - \mu_1 m_u' & \mu_1 m_\eta' \end{pmatrix} \\ &= TAs^3 + TBs^2 + TCs + TD \end{aligned}$$

Now, we have;

Using partial fractions to obtain a form that has a Laplace inverse:

$$\begin{aligned} G_\eta^u(s) &= \frac{\Delta_\eta^u}{\Delta_{long}} = \frac{UAs^3 + UBs^2 + UCs + UD}{(s - s_1)(s - s_1^*)(s - s_3)(s - s_3^*)} \\ &= \frac{A}{s - s_1} + \frac{A^*}{s - s_1^*} + \frac{B}{s - s_3} + \frac{B^*}{s - s_3^*} \\ G_\eta^w(s) &= \frac{\Delta_\eta^w}{\Delta_{long}} = \frac{WAs^3 + WBs^2 + WCs + WD}{(s - s_1)(s - s_1^*)(s - s_3)(s - s_3^*)} \\ &= \frac{A}{s - s_1} + \frac{A^*}{s - s_1^*} + \frac{B}{s - s_3} + \frac{B^*}{s - s_3^*} \\ G_\eta^\theta(s) &= \frac{\Delta_\eta^\theta}{\Delta_{long}} = \frac{TAs^3 + TBs^2 + TCs + TD}{(s - s_1)(s - s_1^*)(s - s_3)(s - s_3^*)} \\ &= \frac{A}{s - s_1} + \frac{A^*}{s - s_1^*} + \frac{B}{s - s_3} + \frac{B^*}{s - s_3^*} \end{aligned}$$

The complex partial fraction coefficients A and B are determined by multiplying and equating coefficients.

Lateral Transfer Functions :

$$G_{(\xi, \epsilon)}^v(s) = \frac{\Delta_{(\xi, \epsilon)}^v}{\Delta_{lat}}, \quad G_{(\xi, \epsilon)}^\phi(s) = \frac{\Delta_{(\xi, \epsilon)}^\phi}{\Delta_{lat}}, \quad G_{(\xi, \epsilon)}^\psi(s) = \frac{\Delta_{(\xi, \epsilon)}^\psi}{\Delta_{lat}}$$

The numerator of the transfer functions are determined using Cramer's Rule : (see Ref 4. pp318-21)

$$\begin{aligned} \Delta_{(\xi, \epsilon)}^v &= \det \begin{pmatrix} y_{(\xi, \epsilon)} & -\frac{y_x}{\mu_2} s + k & (1 - \frac{y_x}{\mu_2})s + k' \\ \frac{\mu_2}{i_A} l_{(\xi, \epsilon)} & s^2 - \frac{l_x}{i_A} s & -\frac{i_E}{i_A} s^2 - \frac{l_x}{i_A} s \\ \frac{\mu_2}{i_v} n_{(\xi, \epsilon)} & -\frac{i_E}{i_v} s^2 - \frac{n_x}{i_v} s & s^2 - \frac{n_x}{i_v} s \end{pmatrix} \\ &= VAs^4 + VBs^3 + VCs^2 + VDs \\ \Delta_{(\xi, \epsilon)}^\phi &= \det \begin{pmatrix} s - y_v & y_{(\xi, \epsilon)} & (1 - \frac{y_x}{\mu_2})s + k' \\ -\frac{\mu_2}{i_A} l_v & \frac{m\mu_2}{i_A} l_{(\xi, \epsilon)} & -\frac{i_E}{i_A} s^2 - \frac{l_x}{i_A} s \\ \frac{\mu_2}{i_v} n_v & \frac{\mu_2}{i_v} n_{(\xi, \epsilon)} & s^2 - \frac{n_x}{i_v} s \end{pmatrix} \\ &= RAs^3 + RBs^2 + RCs + RD \\ \Delta_{(\xi, \epsilon)}^\psi &= \det \begin{pmatrix} s - y_v & -\frac{y_x}{\mu_2} s + k & y_{(\xi, \epsilon)} \\ -\frac{\mu_2}{i_A} l_v & s^2 - \frac{l_x}{i_A} s & \frac{m\mu_2}{i_A} l_{(\xi, \epsilon)} \\ \frac{\mu_2}{i_v} n_v & \frac{i_E}{i_v} s^2 - \frac{n_x}{i_v} s & \frac{\mu_2}{i_v} n_{(\xi, \epsilon)} \end{pmatrix} \\ &= YAs^3 + YBs^2 + YCs + YD \end{aligned}$$

Now, we have;

Using partial fractions to obtain a form that has a Laplace inverse:

$$\begin{aligned} AA.G_{(\xi, \epsilon)}^v &= AA. \frac{\Delta_{(\xi, \epsilon)}^v}{\Delta_{lat}} = \frac{VAs^4 + VBs^3 + VCs^2 + VDs}{s(s-s_1)(s-s_2)(s-s_3)(s-s_3^*)} \\ &= \frac{B_1}{s-s_1} + \frac{B_2}{s-s_2} + \frac{C}{s-s_3} + \frac{C^*}{s-s_3^*} \\ AA.G_{(\xi, \epsilon)}^\phi &= AA. \frac{\Delta_{(\xi, \epsilon)}^\phi}{\Delta_{lat}} = \frac{RAs^3 + RBs^2 + RCs + RD}{s(s-s_1)(s-s_2)(s-s_3)(s-s_3^*)} \\ &= \frac{B_1}{s-s_1} + \frac{B_2}{s-s_2} + \frac{C}{s-s_3} + \frac{C^*}{s-s_3^*} + \frac{D}{s} \\ AA.G_{(\xi, \epsilon)}^\psi &= AA. \frac{\Delta_{(\xi, \epsilon)}^\psi}{\Delta_{lat}} = \frac{YAs^3 + YBs^2 + YCs + YD}{s(s-s_1)(s-s_2)(s-s_3)(s-s_3^*)} \\ &= \frac{B_1}{s-s_1} + \frac{B_2}{s-s_2} + \frac{C}{s-s_3} + \frac{C^*}{s-s_3^*} + \frac{D}{s} \end{aligned}$$

The real coefficients B_1 and B_2 and D and the complex coefficient C are determined multiplying out and equating coefficients.

A2.5 Unit Impulse Response Functions

Unit Impulse Response functions analytically define the response of any parameter to the unit impulse input of any other input. That is;

$$h_Y^X = \text{Response of any parameter, } X, \text{ to a unit impulse input of } Y.$$

Taking the inverse Laplace transform of the transfer function yields the Unit Impulse Response function :

$$h_Y^X(\tau) = \mathcal{L}^{-1} G_Y^X(s) \cdot Y(s)$$

where

$$Y(s) = \mathcal{L}^{-1}[\text{Unit Impulse}] = 1$$

Longitudinal Unit Impulse Response Functions :

$$\begin{aligned} h_{\eta}^{(\dot{u}, \dot{w}, \theta)}(\tau) &= \mathcal{L}^{-1} G_{\eta}^{(u, w, \theta)} = A e^{s_1 \tau} + A^* e^{s_1^* \tau} + B e^{s_3 \tau} + B^* e^{s_3^* \tau} \\ &= 2|A| e^{\sigma_s \tau} \cos(\omega_s \tau + \angle A) + 2|B| e^{\sigma_s \tau} \cos(\omega_p \tau + \angle B) \end{aligned}$$

See Appendix 2.1 for the above manipulation of h_{η}^X .

Lateral Unit Impulse Response Functions :

$$h_{(\xi, \zeta)}^{(\dot{\psi}, \phi, \psi)}(\tau) = \mathcal{L}^{-1} G_{(\xi, \zeta)}^{(\dot{\psi}, \phi, \psi)} = \frac{B_1}{AA} e^{\sigma_1 \tau} + \frac{B_2}{AA} e^{\sigma_2 \tau} + 2 \frac{|C|}{AA} e^{\sigma_3 \tau} \cos(\omega_3 \tau + \angle C) + \frac{D}{AA}$$

See Appendix 2.1 for the above manipulation of $h_{(\xi, \zeta)}^X$.

(Note : $D \neq 0$ for $h_{(\xi, \zeta)}^{\psi}$.)

A2.6 Atypical Eigen Analysis Conditions

If the user doesn't request the eigen analysis to be performed then only the eigen values for the system are determined and any non-normal (meaning not the type of result expected for a conventional aircraft) conditions are reported to the user.

The atypical conditions include the following;

- Characteristic equations
 - s^0 coefficient in Long-CE not equal to zero.
 - s^0 coefficient in Lat-CE not equal to zero.
- Real roots for Short Period Oscillation mode.
- Positive real part (unstable) of SPO mode.
- Real roots for Phugoid mode.
- Positive real part (unstable) of Phugoid mode.
- Complex conjugate root for Roll and Spiral modes.
- Real roots for Dutch Roll mode.
- Positive real part (unstable) of Dutch Roll mode.

APPENDIX 2.1 : Manipulation of $h(\tau)$

$$\begin{aligned} h(\tau) &= Ae^{(\sigma + i\omega)\tau} + A^*e^{(\sigma - i\omega)\tau} \\ &= Ae^{\sigma\tau}e^{i\omega\tau} + A^*e^{\sigma\tau}e^{-i\omega\tau} \end{aligned}$$

Using : $e^{ix} = \cos x + i \sin x$

$$\begin{aligned} h(\tau) &= Ae^{\sigma\tau}(\cos \omega\tau + i \sin \omega\tau) + A^*e^{\sigma\tau}(\cos(-\omega\tau) + i \sin(-\omega\tau)) \\ &= e^{\sigma\tau}[A(\cos \omega\tau + i \sin \omega\tau) + A^*(\cos \omega\tau - i \sin \omega\tau)] \\ &= e^{\sigma\tau}[(A + A^*)\cos \omega\tau + i(A - A^*)\sin \omega\tau] \end{aligned}$$

$$\begin{aligned} \text{now let } A &= \hat{A} + i\hat{A} \\ A^* &= \hat{A} - i\hat{A} \end{aligned}$$

$$\begin{aligned} \text{so } A + A^* &= 2\hat{A} \\ \text{and } A - A^* &= i2\hat{A} \end{aligned}$$

therefore

$$\begin{aligned} h(\tau) &= 2e^{\sigma\tau}[\hat{A}\cos \omega\tau - \hat{A}\sin \omega\tau] \\ &= 2|A|e^{\sigma\tau}\left[\frac{\hat{A}}{|A|}\cos \omega\tau - \frac{\hat{A}}{|A|}\sin \omega\tau\right] \end{aligned}$$

where $|A|$ is the magnitude of the complex number A

$$\text{also, } \cos \angle A = \frac{\hat{A}}{|A|}$$

$$\text{and } \sin \angle A = \frac{\hat{A}}{|A|}$$

($\angle A$ is the argument of the complex number A)

therefore

$$h(\tau) = 2|A|e^{\sigma\tau}[\cos \angle A \cos \omega\tau - \sin \angle A \sin \omega\tau]$$

Now, using : $\cos x \cos y - \sin x \sin y = \cos(x + y)$

gives

$$h(\tau) = 2|A|e^{\sigma\tau} \cos(\omega\tau + \angle A)$$

APPENDIX 3 : Aircraft Forces and Moments

The forces and moments acting on the aircraft are represented as a steady state component together with components due to perturbations about the reference steady state, taken to be symmetric wings level (but not necessarily horizontal) translational flight.

A3.1 Forces

The forces (X, Y, Z) each consist of an aerodynamic component (X_a, Y_a, Z_a) and a gravitational component (X_g, Y_g, Z_g) .

Aerodynamic Forces

$$X_a = X_{a_0} + uX_u + wX_w + qX_q + X(t)$$

$$Y_a = Y_{a_0} + vY_v + pY_p + rY_r + Y(t)$$

$$Z_a = Z_{a_0} + uZ_u + wZ_w + qZ_q + Z(t)$$

The steady state values are :

$$X_{a_0} = mg \sin \Theta_0$$

$$Y_{a_0} = 0$$

$$Z_{a_0} = -mg \cos \Theta_0$$

The time varying components for this application are due to the control surfaces only (time varying excess thrust is not be incorporated but could be if necessary) :

$$X(t) = |\eta(t)| X_\eta$$

$$Y(t) = \xi(t) Y_\xi + \zeta(t) Y_\zeta$$

$$Z(t) = \eta(t) Z_\eta$$

Gravitational Forces

Because the Flight Profile Generator program keeps track of the aircraft attitude via a direction cosine matrix (DCM), the need for AcSim to calculate the angular displacements of the aircraft is dispensed with and the gravity vector components are determined using the DCM sent in from FPG2 at every time step.

The gravitational forces are;

$$\begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix}_{Body} = C_B^N \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

where C_B^N is the transformation (direction cosine) matrix from Navigation axes (local North, East, Down) to aircraft body axes. The direction cosine matrix from Body-to-Nav axes is calculated in FPG2 and is sent into AcSim, and since $C_B^N = (C_N^B)^T$ we obtain :

$$X_g = C_B^N(1,3)mg = C_N^B(3,1)mg$$

$$Y_g = C_B^N(2,3)mg = C_N^B(3,2)mg$$

$$Z_g = C_B^N(3,3)mg = C_N^B(3,3)mg$$

A3.2 Moments

The moments (L, M, N) consist of aerodynamic components (L_a, M_a, N_a) only. The moments due to gravity (L_g, M_g, N_g) are zero since the body axes origin was chosen as the aircraft's center of gravity.

Aerodynamic Moments

$$L_a = L_{a_0} + vL_v + pL_p + rL_r + L(t)$$

$$M_a = M_{a_0} + uM_u + wM_w + \dot{w}M_{\dot{w}} + qM_q + M(t)$$

$$N_a = N_{a_0} + vN_v + pN_p + rN_r + N(t)$$

The steady state values $(L_{a_0}, M_{a_0}, N_{a_0})$ are zero, since the aircraft is initially in steady wings-level trimmed flight.

The time varying components are:

$$L(t) = \xi(t)L_\xi + \zeta(t)L_\zeta$$

$$M(t) = \eta(t)M_\eta$$

$$N(t) = \xi(t)N_\xi + \zeta(t)N_\zeta$$

So the force and moment equations, in stability axes, are :

$$X = C_N^B(3, 1)mg + mg \sin \Theta_0 + (U(t) - V_0)X_u + W(t)X_w + q(t)X_q + \eta(t)X_\eta$$

$$Y = C_N^B(3, 2)mg + V(t)Y_v + p(t)Y_p + r(t)Y_r + \xi(t)Y_\xi + \zeta(t)Y_\zeta$$

$$Z = C_N^B(3, 1)mg - mg \cos \Theta_0 + (U(t) - V_0)Z_u + W(t)Z_w + q(t)Z_q + \eta(t)Z_\eta$$

$$L = V(t)L_v + p(t)L_p + r(t)L_r + \xi(t)L_\xi + \zeta(t)L_\zeta$$

$$M = (U(t) - V_0)M_u + w(t)M_w + \dot{w}(t)M_{\dot{w}} + q(t)M_q + \eta(t)M_\eta$$

$$N = V(t)N_v + p(t)N_p + r(t)N_r + \xi(t)N_\xi + \zeta(t)N_\zeta$$

APPENDIX 4 : Runge-Kutta Implementation

FPG2 uses a fourth order Runge-Kutta integration method, which requires the aircraft state data (linear accelerations and angular velocities) at the beginning, twice at the middle, and at the end of each time step. In order to achieve this AcSim also uses a fourth order Runge-Kutta scheme is implemented as follows ;

At $t = t_0$
 get $\dot{S}\hat{V}_{(t_0)}$ using $S\hat{V}_{(t_0)}$
 $\hat{k}_1 = \Delta t \cdot \dot{S}\hat{V}_{(t_0)}$
 return to FPG2 with $S\hat{V}_{(t_0)}$ and $\dot{S}\hat{V}_{(t_0)}$

At $t = t_0 + \frac{1}{2}\Delta t$
 get $S\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(1)}} = S\hat{V}_{(t_0)} + \frac{1}{2}\hat{k}_1$
 get $\dot{S}\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(1)}}$ using $S\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(1)}}$
 $\hat{k}_2 = \Delta t \cdot \dot{S}\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(1)}}$
 return to FPG2 with $S\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(1)}}$ and $\dot{S}\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(1)}}$

At $t = t_0 + \frac{1}{2}\Delta t$
 get $S\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(2)}} = S\hat{V}_{(t_0)} + \frac{1}{2}\hat{k}_2$
 get $\dot{S}\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(2)}}$ using $S\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(2)}}$
 $\hat{k}_3 = \Delta t \cdot \dot{S}\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(2)}}$
 return to FPG2 with $S\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(2)}}$ and $\dot{S}\hat{V}_{(t_0 + \frac{1}{2}\Delta t)^{(2)}}$

At $t = t_0 + \Delta t$
 get $S\hat{V}_{(t_0 + \Delta t)} = S\hat{V}_{(t_0)} + \hat{k}_4$
 get $\dot{S}\hat{V}_{(t_0 + \Delta t)}$ using $S\hat{V}_{(t_0 + \Delta t)}$
 $\hat{k}_4 = \Delta t \cdot \dot{S}\hat{V}_{(t_0 + \Delta t)}$
 return to FPG2 with $S\hat{V}_{(t_0 + \Delta t)}$ and $\dot{S}\hat{V}_{(t_0 + \Delta t)}$
 and update :
 $t_1 = t_0 + \Delta t$
 $S\hat{V}_{(t_1)} = S\hat{V}_{(t_0)} + \frac{1}{6}(\hat{k}_1 + 2\hat{k}_2 + 2\hat{k}_3 + \hat{k}_4)$

APPENDIX 5 : Numerical Convolution Solution

Once all the unit impulse response (UIR) functions and their derivatives are calculated the response to the given control deflections (forcing function) is found using a numerical integration technique: numerical convolution.

The convolution theorem (ref. 4) is:

If $f(t)$ and $g(t)$ are the inverse Laplace transforms of $F(s)$ and $G(s)$, respectively, then the inverse transform, $h(t)$, of the product $H(s) = F(s)G(s)$ is the convolution of $f(t)$ and $g(t)$, and is defined by

$$h(t) = \int_0^t f(\tau)g(t-\tau)d\tau$$

The algorithm for numerical convolution is:

$$x_i = \Delta t \sum_{j=1}^{i-1} (f_j h_{i-j})$$

where,

- $t = (i-1)\Delta t$ time at center of i^{th} interval
- x_i value of response at center of i^{th} interval
- f_i value of input function at center of i^{th} interval
- h_i value of UIR at the end of i^{th} interval
- j number of intervals forcing function is divided into.

Note that in the above algorithm $f = f(t)$, and $h = h(t)$, but in the eigen analysis theory the time is non-dimensionalised ($\tau = \frac{t}{t_n}$), hence we obtain $h = h(\tau)$. Therefore we must use $f = f(\tau)$ and Δt must be replaced by $\Delta \tau = \frac{\Delta t}{t_n}$.

We can think of this method as using the UIRs as influence coefficients, that is, the response, say at $t = t_n$, is the summation of the forcing function at $t = t_i$ for $0 < t_i < t_n$, multiplied by the UIR, with the UIRs origin shifted to $t = t_i$ (so that the time between the application of the force, at t_i , and the response, at t_n , to that force is $t_n - t_i$). In order for this to be done numerically the forcing function is divided into intervals of width Δt .

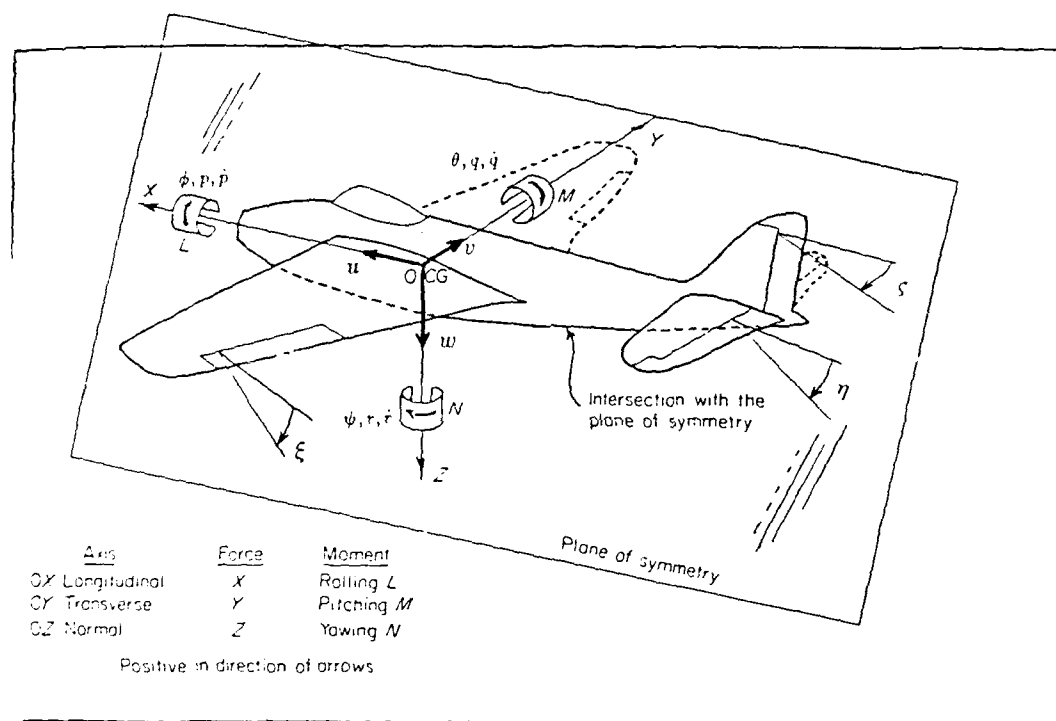
The elements of the forcing function are considered impulses of magnitude $f(t_i)\Delta t$ acting at the center of the interval. So, the response, say X , at $t = t_n$ is found from

$$X(t_n) = f(t_1)h_{f(t_1)}^*(t - t_1)\Delta t + f(t_2)h_{f(t_2)}^*(t - t_2)\Delta t + \dots + f(t_n)h_{f(t_n)}^*(t - t_n)\Delta t$$

When responses are calculated for parameters with UIRs derived from other UIRs, an extra term must be added to the results during the application of the control deflection. To demonstrate this, consider $db\dot{v}/dt$

$$\begin{aligned} \frac{db\dot{v}}{dt} &= \int_0^t h_{\eta}^{w'}(t-\tau)\eta(t)d\tau \\ &= \int_0^t \frac{d}{dt} [h_{\eta}^w(t-\tau)]\eta(t)d\tau + \int_0^t \delta(t-\tau)h_{\eta}^w(t-\tau)\eta(t)d\tau \\ &= \int_0^t \frac{d}{dt} [h_{\eta}^w(t-\tau)]\eta(t)d\tau + h_{\eta}^w(0)\eta(t) \end{aligned}$$

FIGURE 1 : Aircraft axes and notation



* Figure from Ref. 2, p607

FIGURE 2 : AcSim Program Summary Flowchart

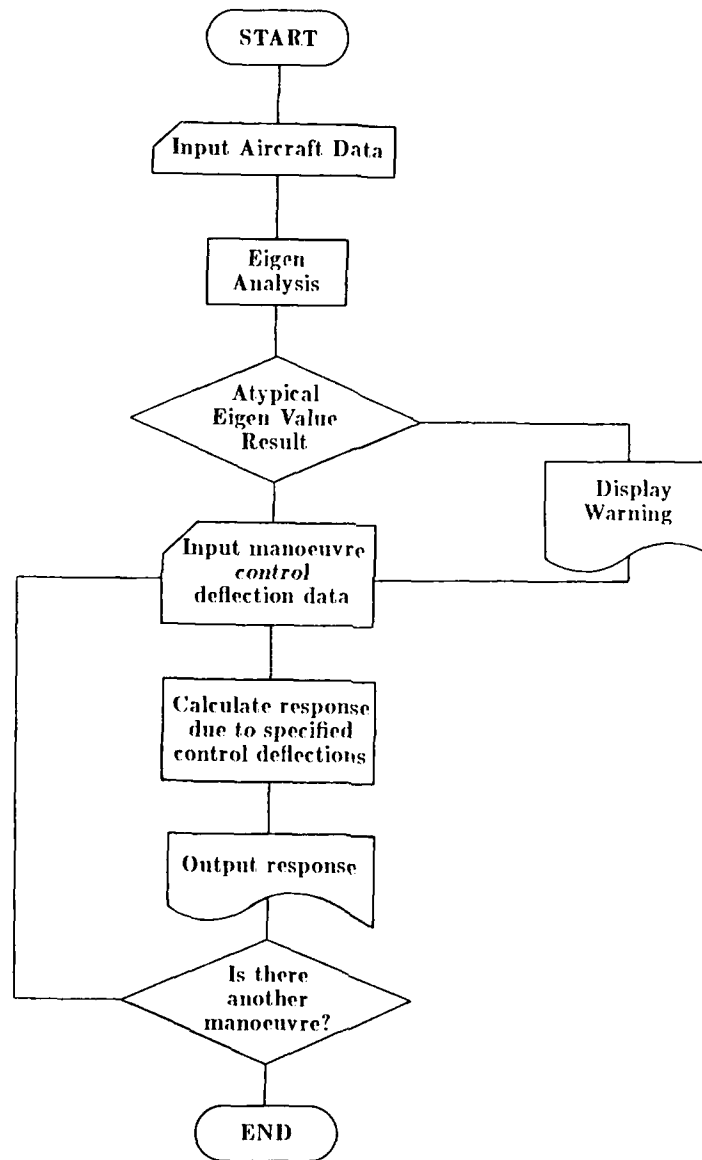


FIGURE 3 : AcSim Program Routine Structure

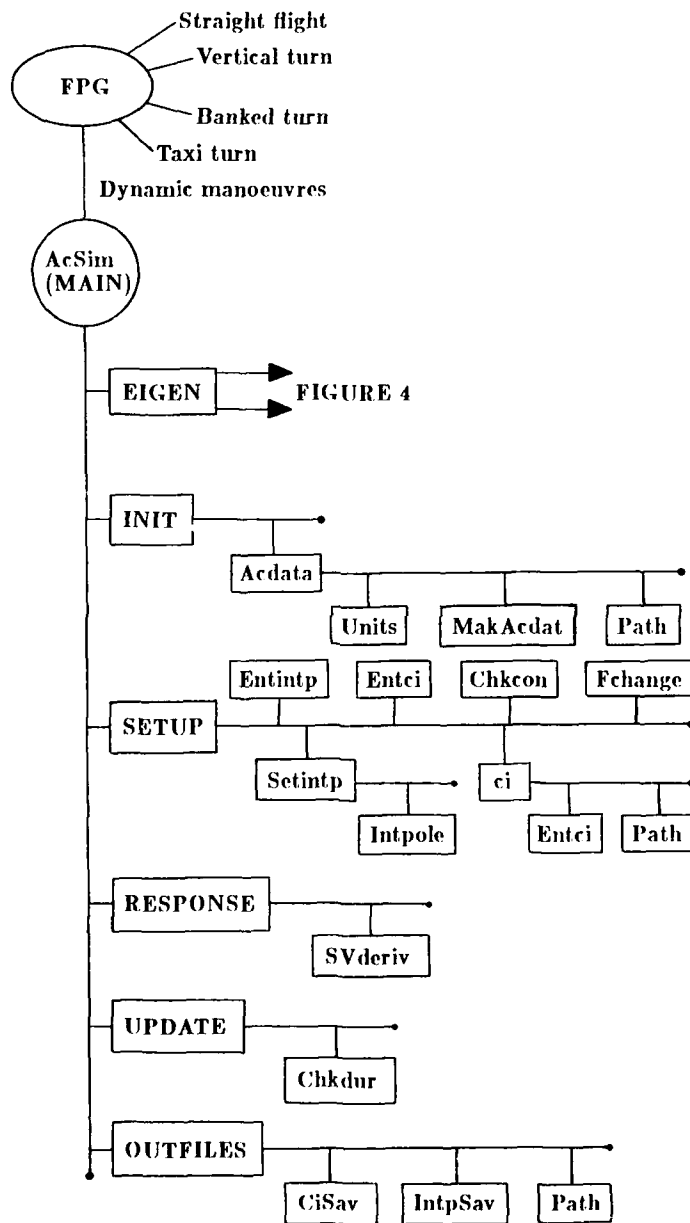


FIGURE 4 : Eigen Analysis Subroutine Group

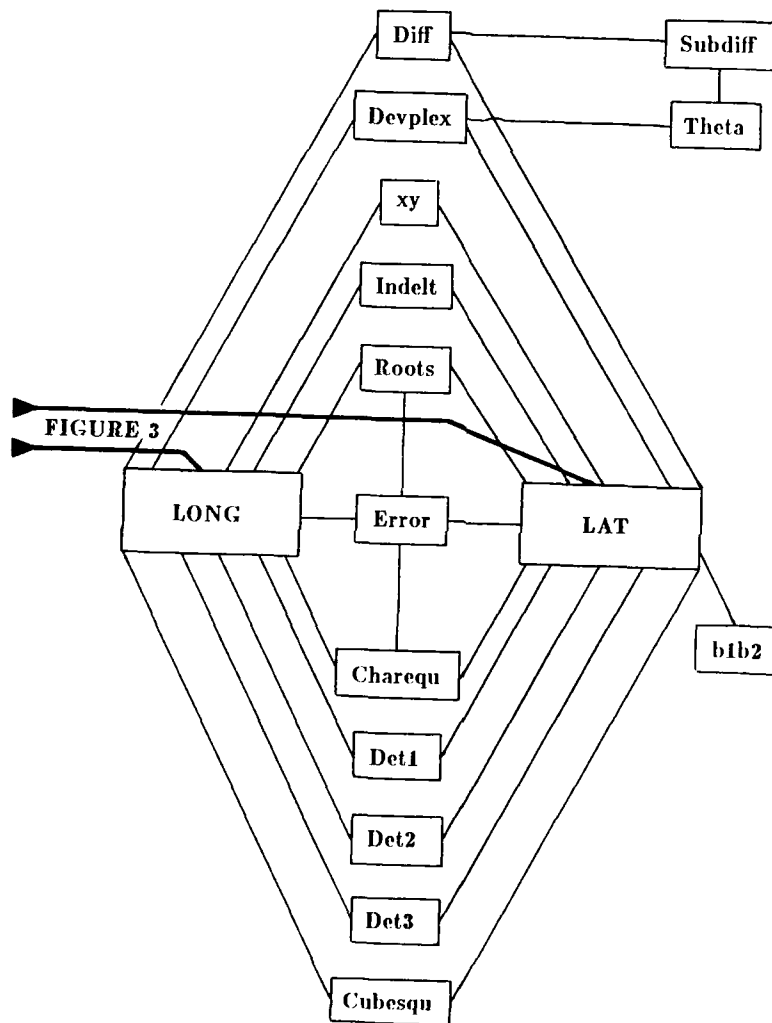


FIGURE 5 : Bonanza Data Summary File

*** A I R C R A F T D A T A F I L E ***

Air Density = .002000 slugs/ft^3
Aircraft Mass = 3000.000000 lbs
Moments of Inertia :
 About Roll (x) axis = Ixx = 1200.000000
 About Pitch (y) axis = Iyy = 2000.000000
 About Yaw (z) axis = Izz = 3000.000000
Products of Inertia :
 About x and z axes = Ixz = 100.000000
Wing Planform Area = 181.000000 ft^2
Wing Span = 33.500000 ft
Tailplane lever arm = 15.200000 ft
 (aircraft c.g to tailplane aerodynamic center)

** LONGITUDINAL Non-Dimensional Aerodynamic Derivatives

xu = -0.019 xw = 0.056 xq = 0
zu = -0.197 zw = -2.12 zq = -0.269
mu = 0 mw = -0.306 mq = -0.356
 mw dot = -0.0083

x eta = 0
z eta = -0.4
m eta = -0.43

** LATERAL Non-Dimensional Aerodynamic Derivatives

yv = -0.32 yp = 0 yr = 0
lv = -0.06 lp = -0.35 lr = 0.028
nv = 0.081 np = -0.039 nr = -0.123

y ail = 0 y rud = 0.02
l ail = 0.02 l rud = 0.005
n ail = -0.003 n rud = -0.1

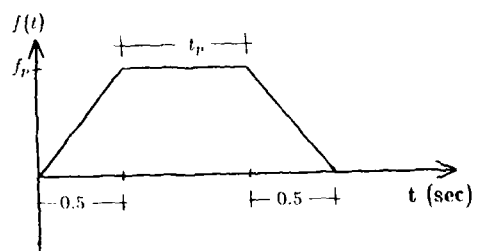
FIGURE 6 : Partial Eigen Analysis Results File

```

*****
**                               Eigen Analysis Results                               **
**-----**
** Flight Condition : Speed = 290.000 ft/sec **
**                               t-hat = .889 **
**-----**
** LONGITUDINAL : **
**-----**
** Characteristic Equation = 1.0000 s^4 **
**                               + 7.4676 s^3 **
**                               + 63.3017 s^2 **
**                               + 1.2716 s **
**                               + 1.0852 **
** Short Period Oscillation : (Complex roots) **
**                               (sigmaS +/- i*omegaS) **
** - Non-dim Damping Factor ,sigmaS = -3.7247 **
** - Non-dim Frequency ,omegaS = 7.0197 **
** Phugoid Oscillation : (Complex roots) **
**                               (sigmaP +/- i*omegaP) **
** - Non-dim Damping Factor ,sigmaP = -.0091 **
** - Non-dim Frequency ,omegaP = .1308 **
**-----**
** LATERAL : **
**-----**
** Characteristic Equation = .9972 s^5 **
**                               + 9.0336 s^4 **
**                               + 21.3877 s^3 **
**                               + 94.4173 s^2 **
**                               + 1.4745 s **
** Roll and Spiral Modes : (Real roots) **
** - Roll damping factor, sigma1 = -7.8596 **
** - Spiral damping factor, sigma2 = -.0157 **
** Dutch Roll Mode : (Complex roots) **
**                               (sigma3 +/- i*omega3) **
** - Non-dim Damping Factor, sigma3 = -.5917 **
** - Non-dim Frequency, omega3 = 3.4138 **
*****

```

FIGURE 7 : Control Deflection Schedule



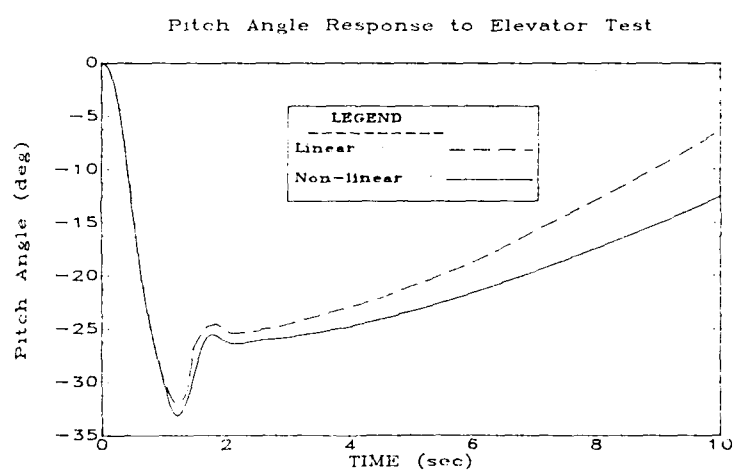
$f(t)$ = control surface deflection (degrees)

= $\eta(t), \xi(t), \zeta(t)$

f_p = peak deflection = 10°

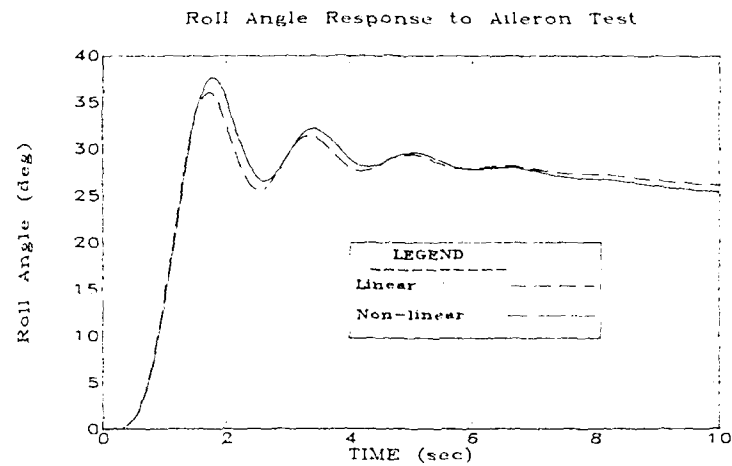
t_p = duration of peak = 0.5 sec.

FIGURE 8 : Validation Test Runs

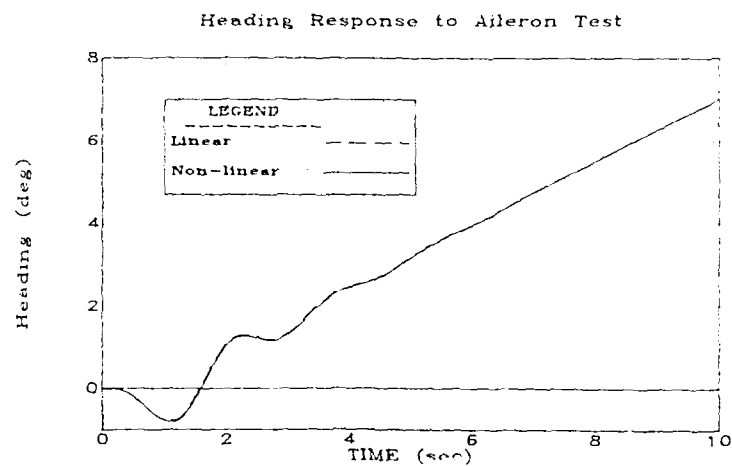


(a) Pitch Angle Response to Elevators

FIGURE 8 : Validation Test Runs (cont.)

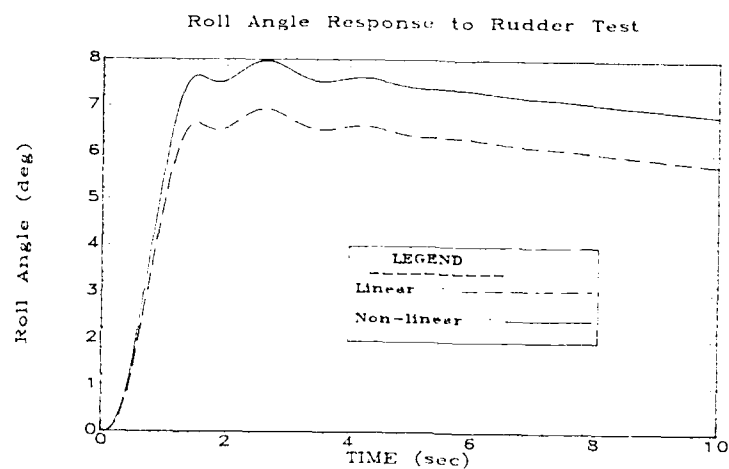


(b) Roll Angle Response to Ailerons

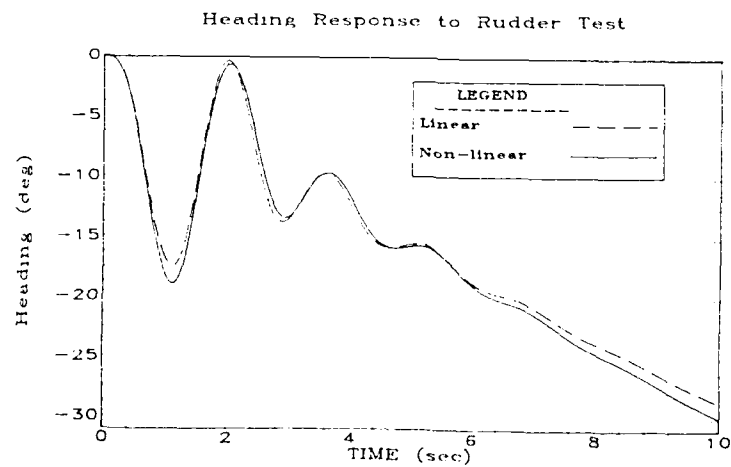


(c) Heading Response to Ailerons

FIGURE 8 : Validation Test Runs (cont.)



(d) Roll Angle Response to Rudder



(e) Heading Response to Rudder

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16. ABSTRACT Program AcSim was designed to be used in conjunction with a flight profile generator program (FPG2) which is used with an Inertial Navigation System simulator. Both AcSim and FPG2 were written in FORTRAN-77 on an ELXSI System 6400. AcSim is implemented as a subroutine of FPG2 and is called if the user selects a dynamic manoeuvre segment to be performed. AcSim calculates the dynamic state of an aircraft for a sequence of user-specified control surface deflections. The aircrafts linear accelerations and angular velocities are sent back to FPG2 for subsequent processing.			

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A six-degree-of-freedom model is used to simulate the user-defined aircraft. AcSim implements a State Vector approach to solve the non-linear equations of motion simultaneously using a 4th order Runge-Kutta scheme. An optional "Eigen Analysis" can be performed which provides the user with longitudinal and lateral characteristic equations, their roots (eigen values), transfer functions and unit impulse response functions for the given flight condition.

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